Supporting Information

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SI Text

Here we present a simple model designed to predict existence and value of optimal angle in a biconical channel. Only the case of perfect slip is considered. First, consider a junction between two truncated cones with respective angles α_1 and α_2 (Fig. S1). Far from the junction, the flow is similar to the flow in a cone of infinite extent, and involves no viscous loss. Dissipation is limited to the vicinity of the junction. What is the corresponding hydrodynamic resistance?

An exact calculation, along the lines of ref. 1, would involve writing the general solution of the Stokes equation as a series expansion in each domain, and matching them through continuity of velocity and stress. This approach results in a system of equations for the series coefficients that can only be solved numerically. It is then unclear whether additional insight could be gained compared with FE calculations. Instead, we propose an approximate but simple model to capture the angle dependence of the hydrodynamic resistance. We assume an ansatz expression for the flow, based on exact far-field structure, and compute the corresponding pressure drop. This flow field is actually the simplest one can imagine: it is plug-like everywhere, i.e., the velocity is purely radial and uniform on every spherical cap. The pros of such a flow is that it is correct far from the junction, and that it satisfies velocity continuity and the perfect-slip boundary condition (BC). The cons are that it is not a solution of the Stokes equation, and involves a nonvanishing velocity at the corner. With notations given in Fig. S1, the velocity is $v = \frac{q}{2\pi a^2}(1+\cos\alpha')\mathbf{e}'_r$, for all $\alpha' \in [\alpha_1, \alpha_2]$. Pressure drop along the z axis occurs only in the transition zone and can be evaluated using $\nabla p = \eta \Delta \mathbf{v}$, leading to a simple expression for the resistance:

$$\frac{\Delta p}{Q} = g(\alpha_1, \alpha_2, a) = f(\alpha_2, a) - f(\alpha_1, a), \qquad \textbf{[S1]}$$

$$f(\alpha, a) = \frac{\eta}{2\pi a^3}[2\alpha + \sin\alpha(1-\cos\alpha)].$$

For the particular case $\alpha_1 = 0$ and $\alpha_2 = \frac{\pi}{2}$, this equation predicts a resistance of $0.66\eta/a^3$. Given that the actual prefactor, as obtained from finite-element (FE) calculation, is $3.75/2 = 1.875$, we see that the resistance is grossly underestimated. Our only claim is to capture the angle dependence. It is now straightforward to model the biconical channel, as two junctions in series, with a total resistance

$$R = g(0, \alpha, a) + g(\alpha, \pi/2, a + L\ \tan\alpha). \qquad \textbf{[S2]}$$

Fig. S2 shows this resistance as a function of α for several cone lengths. Interestingly, all curves have a minimum (providing a large gain in resistance), and the optimal angle, although not quantitatively predicted (50% discrepancy), shows the correct dependence in L.

1. Dagan Z, Weinbaum S, Pfeffer R (1982) An infinite-series solution for the creeping motion through an orifice of finite length. *J Fluid Mech* 115:505–523.

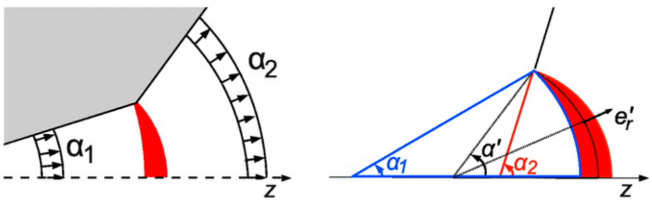

Fig. S1. Schematic of the junction between two truncated cones. Perfect-slip BC is assumed. The transition zone is highlighted in red. $\mathbf{e}'_r$ is the radial unit vector for a given $\alpha' \in [\alpha_1, \alpha_2]$.

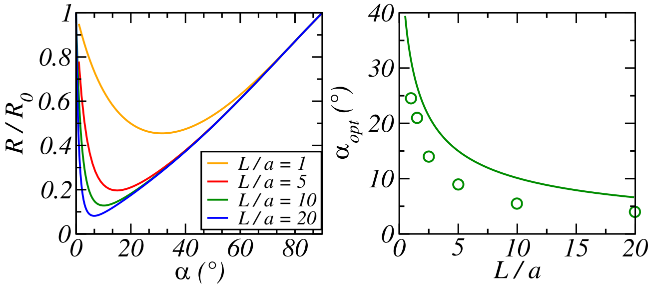

Fig. S2. Predictions of the approximate simple model. (*A*) Total hydrodynamic resistance R of the channel as a function of the angle α obtained for different values of L/a. R is normalized with respect to $R_0 = R(\alpha = 0)$. (*B*) Optimal angle α_{opt} as a function of cone length L, from the model (line) and from FE calculations (points).